AP Calculus BC — Integration by Substution

1) $\int \cos(6x)\,dx$

$u = 6x$

$\frac{du}{dx} = 6$

$du = 6\,dx$

$\frac{1}{6}du = dx$

$\frac{1}{6}\int \cos u\,du$

$\boxed{\frac{1}{6}\sin u + C}$

2) $\int 63(9x-7)^{-8}\,dx$

$u = 9x - 7$

$du = 9\,dx$

$\frac{1}{9}du = dx$

$7\int u^{-8}\,du$

$7\,\frac{(u^{-7})}{-7} + C$

$-u^{-7} + C$

$\boxed{-(9x-7)^{-7} + C}$

3) $\int 28r^6(7-r^7)^{-1/2}\,dr$

$u = 7 - r^7$

$du = -7r^6\,dr$

$\frac{du}{-7r^6} = dr$

$\int 28r^6(u)^{-1/2}\,\frac{du}{-7r^6}$

$-4\int u^{-1/2}\,du$

$-4\left[2u^{1/2}\right] + C$

$\boxed{-8(7-r^7)^{1/2} + C}$

4) $\int 12(y^4+4y^2+4)^2\underbrace{(y^3+2y)\,dy}$

$u = y^4 + 4y^2 + 4$

$du = (4y^3 + 8y^2)\,dy$

$\frac{1}{4}du \quad (y^3 + 2y)\,dy$

$\int 12u^2 \cdot \frac{1}{4}\,du$

$3\int u^2\,du$

$3\left[\frac{1}{3}u^3\right] + C$

$\boxed{(y^4+4y^2+4)^3 + C}$

5) $\int \frac{5}{(5x-3)^2}\,dx \rightarrow \int 5(5x-3)^{-2}\,dx$

$u = 5x - 3$

$du = 5\,dx$

$\int u^{-2}\,du$

$-u^{-1} + C$

$\boxed{\frac{-1}{(5x-3)} + C}$

6) $\int \sin(8z-9)\,dz$

$u = 8z - 9$

$du = 8\,dz$

$\frac{1}{8}du = dz$

$\frac{1}{8}\int \sin u\,du$

$-\frac{1}{8}\cos u + C$

$\boxed{-\frac{1}{8}\cos(8z-9) + C}$

7) $\int \frac{\ln^{14}x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u^{14}\,du$

$\frac{1}{15}u^{15} + C$

$\boxed{\frac{1}{15}\ln^{15}x + C}$

8) $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$\int \sin x(\cos x)^{-1}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$-\int u^{-1}\,du$

$-\ln|u| + C$

$\boxed{-\ln|\cos x| + C}$

9) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$\int e^{x^{1/2}} \cdot x^{-1/2}\,dx$

$u = x^{1/2}$ $\quad \frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$du = \frac{1}{2}x^{-1/2} \cdot dx$

$2du = x^{-1/2}dx$

$2\int e^u\,du$

$2e^u + C \rightarrow \boxed{2e^{x^{1/2}} + C}$

10) $\int_4^{19} f(u)\,du = 10$

$\int_1^4 [x \cdot f(x^2+3)]\,dx$

$u = x^2 + 3 \quad du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$u(1) = 4 \quad u(4) = 19$

$\frac{1}{2}\int_4^{19} f(u)\,du = \frac{1}{2}(10) = \boxed{5}$